Introduction to Parallel Algorithm Analysis

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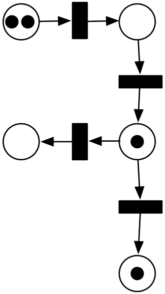
Petri Nets

C. A. Petri [1962] introduced analysis model for concurrent systems.

- Flow chart
- Described data flow and dependencies.
- Very low level (we want something more high-level)
- Reachability EXP-SPACE-HARD, Decidable



Critical Regions Problem

Edsger Dijkstra [1965]

- Mutex: “Mutual exclusion” of variable
- Semaphores : Locks/Unlocks access to (multiple) data.
- Semaphore more general - keeps a count. Mutex binary.

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Important, but lower level details.

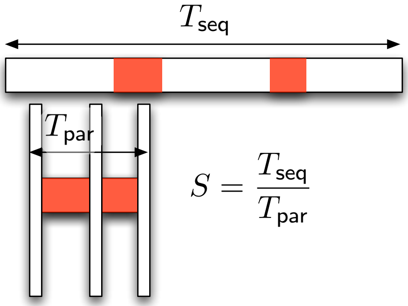
Amdahl's and Gustafson's Laws

Amdahl's Law : Gene Amdahl [1967]

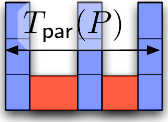
- Small portion (fraction α) non-parallelizable
- Limits max speed-up $S = 1/\alpha$.

Gustafson's Law : Gustafson+Barsis [1988]

- Small portion (fraction α) non-parallelizable
- P processors
- Limits max speed-up $S(P) = P - \alpha(P - 1)$.



$$S = \frac{T_{\text{seq}}}{T_{\text{par}}}$$



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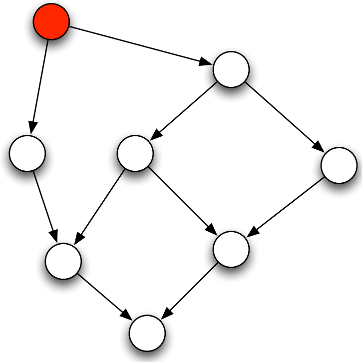
Logical Clocks



Leslie Lamport [1978]

- Posed parallel problems as finite state machine
- Preserved (only) partial order: "happens before" mutex

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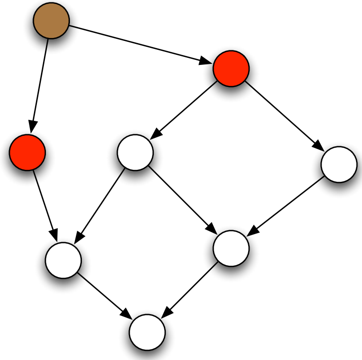

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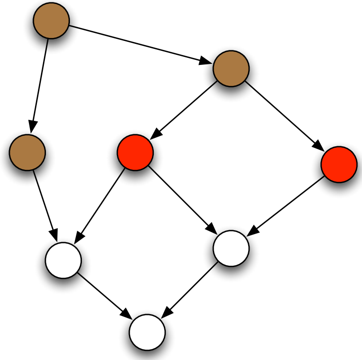

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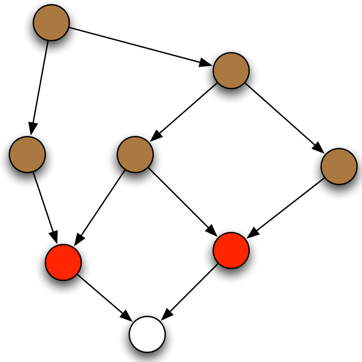

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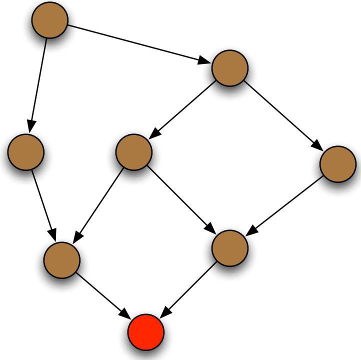

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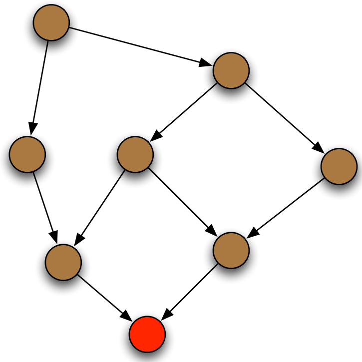

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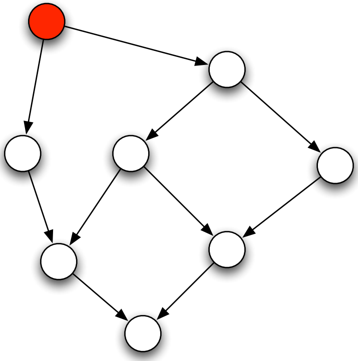
Highlights nuances and difficulties in clock synchronization.

DAG Model

Directed Acyclic Graph:

- Each node represents a chunk of computation that is to be done on a single processor
- Directed edges indicate that the **from** node must be completed before the **to** node
- The longest path in the DAG represents the total amount of parallel time of the algorithm
- The width of the DAG indicates the number of processors that can be used at once



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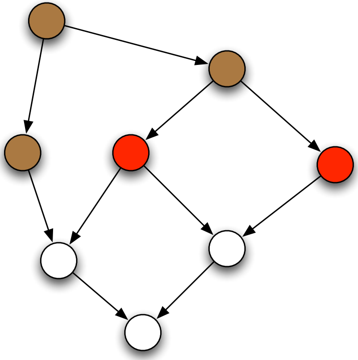

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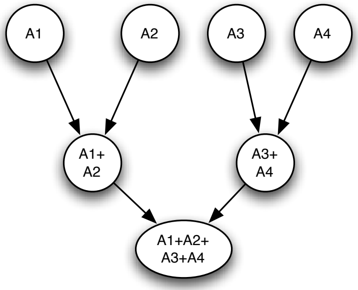
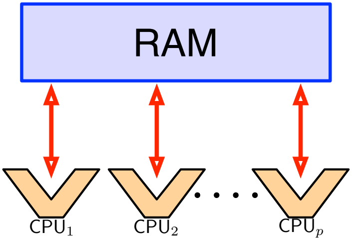


PRAM Model



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"shared memory model"

- P processors which operate on a shared data
- For each processor read, write, op (e.g. $+$, $-$, $\times$) constant time.

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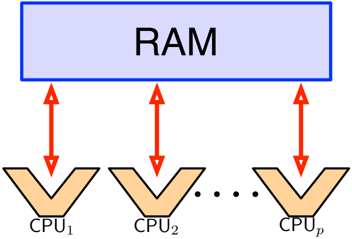

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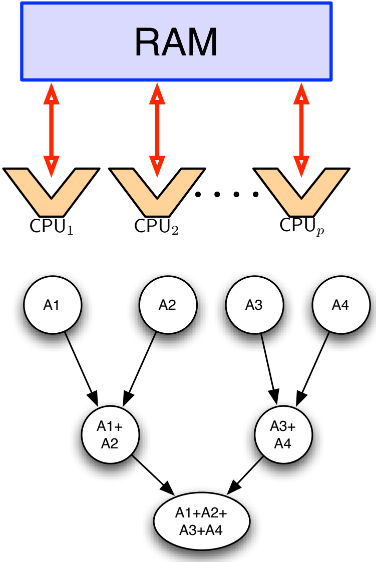

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Emphasizes Locality

- **send**(X, i) : sends X to P_i
- **receive**(Y, j) : receives Y from P_j
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- Array/Ring Topology
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- Mesh Topology
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- Hypercube Topology
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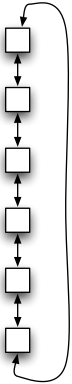

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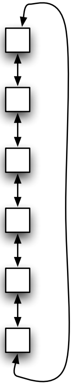

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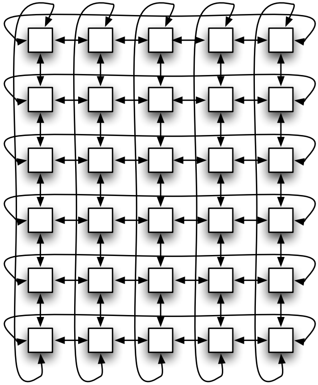

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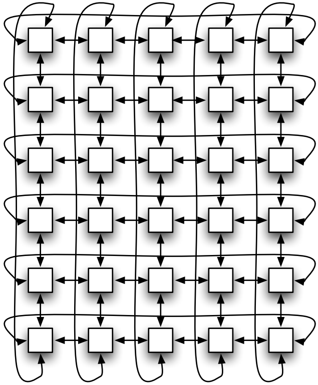

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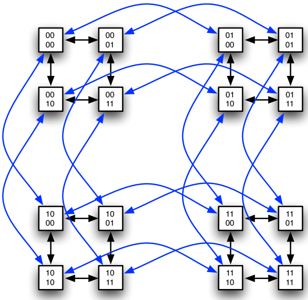

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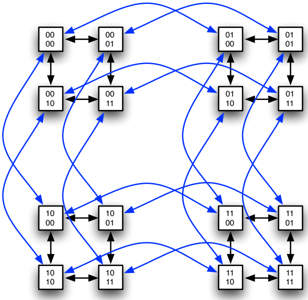

Programming in MPI

Open MPI :

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When to use MPI?

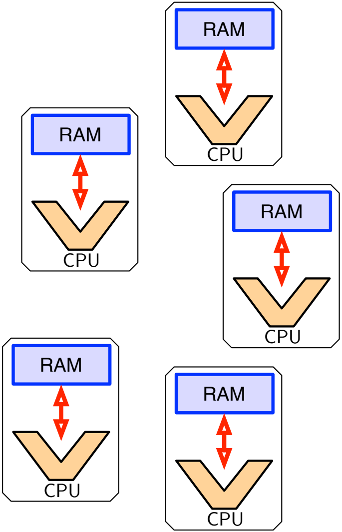
- Critical to exploit locality (i.e. scientific simulations)
- Complication in only talking to neighbor

Bulk Synchronous Parallel

Les Valiant [1989] BSP
Creates "barriers" in parallel algorithm.

1. Each processor computes on data
2. Processors send/receive data
3. Barrier : All processors wait for communication to end globally

Allows for easy synchronization. Easier to analyze since handles many messy synchronization details if this is emulated.



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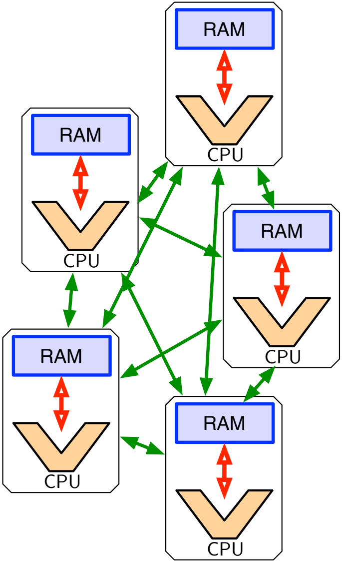

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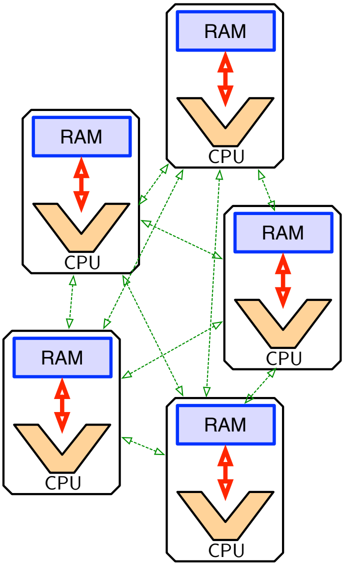

Bridging Model for Multi-Core Computing

Les Valiant [2010] Mulit-BSP
Many parameters:

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- M : Memory/Cache Size
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Argues: any portable and efficient parallel algorithm, must take into account all of these parameters.

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Matrix Multiplication, Fast Fourier Transform, Sorting

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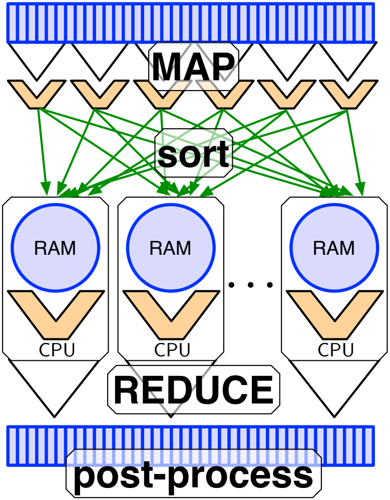
MapReduce

Each Processor has full hard drive, data items $<\text{KEY}, \text{VALUE}>$.
Parallelism Procedes in Rounds:

- Map: assigns items to processor by KEY.
- Reduce: processes all items using VALUE. Usually combines many items with same KEY.

Repeat M+R a constant number of times, often only one round.

- Optional post-processing step.

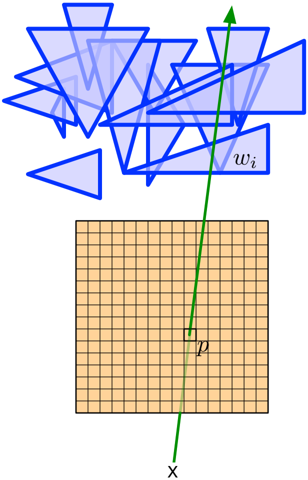


Pro: Robust (duplication) and simple. Can harness Locality
Con: Somewhat restrictive model

General Purpose GPU

Massive parallelism on your desktop.
Uses **G**raphics **P**rocessing **U**nit.
Designed for efficient video rasterizing.
Each *processor* corresponds to pixel p

- depth buffer:
  $D(p) = \min_i ||x - w_i||$
- color buffer: $C(p) = \sum_i \alpha_i \chi_i$
- ...



Pro: Fine grain, massive parallelism. Cheap.
Con: Somewhat restrictive model. Small memory.

… and Beyond

Google Sawzall / Dremel

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Berkeley Spark: Processing in memory

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Massive, Unorganized, Distributed Computing

- Bit-Torrent (distributed hash tables)
- SETI @ Home
- Twitter Storm / Facebook Casandra