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MCMD L8 : Streaming | Heavy Hitters = Approximate Counts

Streaming Algorithms

Stream : A = <a1,a2,...,am>
  ai in [n]  size log n
Compute f(A) in poly(log m, log n) space

Let f_j = |{a_i in A | a_i = j}|

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MAJORITY:  if some f_j > m/2, output j
           else,              output NULL

one-pass requires Omega(min{m,n}) space


Simpler:
FP-MAJORITY:  if some f_j > m/2, output j
              else,              output anything

How good w/ O(log m + log n)  (one counter c + one location l)?
  ...

###########################
c = 0, l = X
for (a_i \in A)
  if (a_i = l) c += 1
  else         c -= 1
  if (c <= 0)  c = 1, l = a_i
return l
###########################

Analysis:  if f_j > m/2, then
  if (l != j) then c decremented at most < m/2 times, but c > m/2
  if (l == j) can be decremented < m/2, but is incremented > m/2
if f_j < m/2 for all j, then any answer ok.

----- another view of analysis ------
Let f_j > m/2, and k = m - f_j.
After s steps, let g_s = unseen elements of index j
               let k_s = unseen elements != index j
               let c_s = c if l!=j, and -c if l==j
Claim:  g_s > c+k_s
  base case (s=0, or even s=1) easily true.
  Inductively 4 cases:
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   a_i = l = j : (g_s decremented, c decremented)
   a_i = l != j: (c incremented, k_s decremented)
   a_i !=l != j: (c decremented, k_s decremented)
   a_i !=l = j : (k_s decremented, maybe c incremented)

Since at the end g_s = k_s = 0, then
     0 > c + 0, implies c < 0, and l==j.
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FREQUENT:  for k, output the set {j : f_j > m/k}
 also hard.

k-FREQUENCY-ESTIMATION:  Build data structure S.
For any j in [n], hat{f}_j = S(j) s.t.
 f_j - m/k <= hat{f}_j <= f_j

aka eps-approximate phi-HEAVY-HITTERS:
   Return all f_j s.t. f_j > phi
   Return no f_j s.t. f_j < phi - eps*m
  (any f_j s.t. phi-eps*m < f_j < phi is ok)


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Misra-Gries Algorithm [Misra-Gries '82]

Solves k-FREQUENCY-ESTIMATION in O(k(log m + log n)) space.

Let C be array of k counters C[1], C[2], ..., C[k]
Let L be array of k locations L[1], L[2], ..., L[k]

############################
Set all C = 0
Set all L = X

for (a_i in A)
  if (a_i in L)  <at index j>
     C[j] += 1
  else           <a_i !in L>
    if (|L| < k)
      C[j] = 1
      L[j] = a_i
    else
      C[j] -= 1 forall j in [k]
  for (j in [k])
     if (C[j] <= 0) set L[j] = X
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############################
On query q in [n]
  if (q in L {L[j]=q}) return hat{f}_q = C[j]
  else                 return hat{f}_q = 0
############################
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Analysis

A counter C[j] representing L[j] = q is only incremented if a_i = q

hat{f}_q <= f_q

If a counter C[j] representing L[j] = q is decremented,
then k-1 other counters are also decremented.
This happens at most m/k times.
A counter C[j] representing L[j] = q is decremented at most m/k times.

f_q - m/k <= hat{f}_q

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How do we get an additive eps-approximate FREQUENCY-ESTIMATION ?
i.e. return hat{f}_q s.t.
|f_q - hat{f}_q| <= eps*m

Set k = 2/eps, return C[j] + (m/k)/2

Space O((1/eps) (log m + log n))

Also:
eps-approximate phi-HEAVY-HITTERS for any phi > m*eps in
space O((1/eps) (log m + log n))

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Can solve k-FREQUENT optimally in two passes w/ O(k(log n + log m)) space.
Run M-G algorithm w/ k counters.
For each stored location, make second pass and count exactly.