# Introduction to I/O Efficient Algorithms (External Memory Model)

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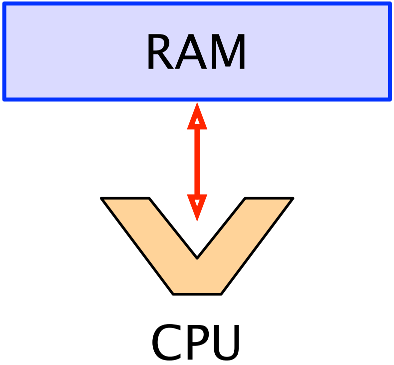
# Von Neumann Architecture

Model:

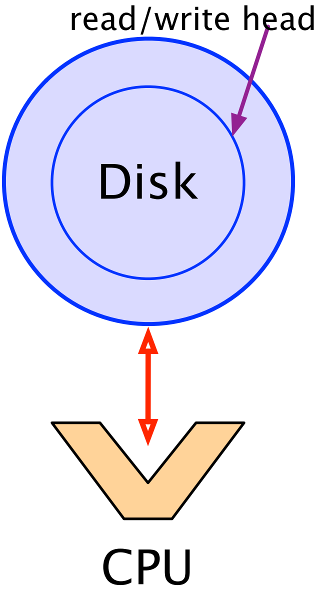
- CPU and Memory
- Read, Write, Operations $(+, -, *, \ldots)$ constant time
- polynomially equivalent to Turing Machine



# Memory as Disk

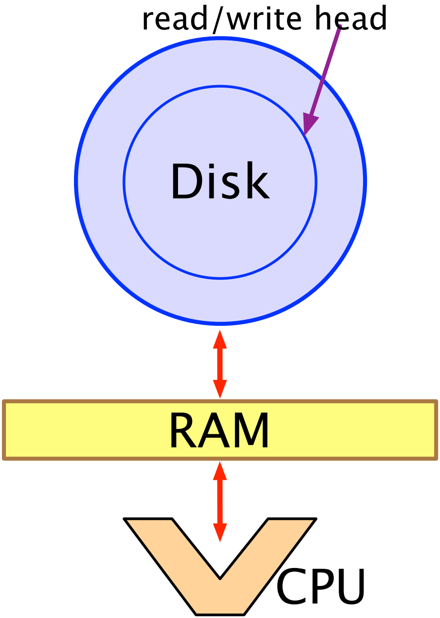
Reality:

- CPU and Memory
- CPU Operations ($+$, $-$, $*$, $\ldots$) constant time
- Read, Write not constant time (at least starting in 1980s).



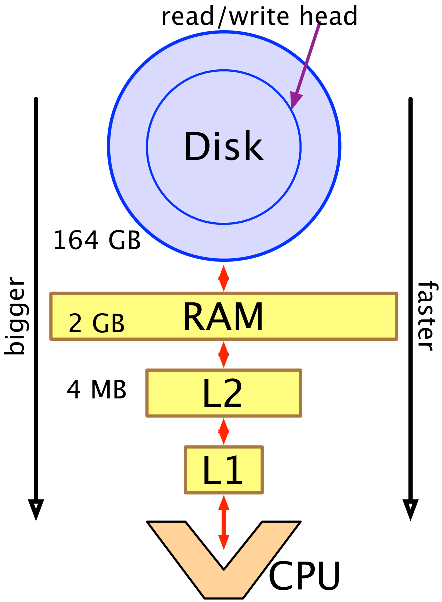
# Cache

- through 1970s: cache access similar to memory access
- First commercially available 1982 (CP/M operating system)
- SmartDrive in Microsoft MS-DOS in 1988



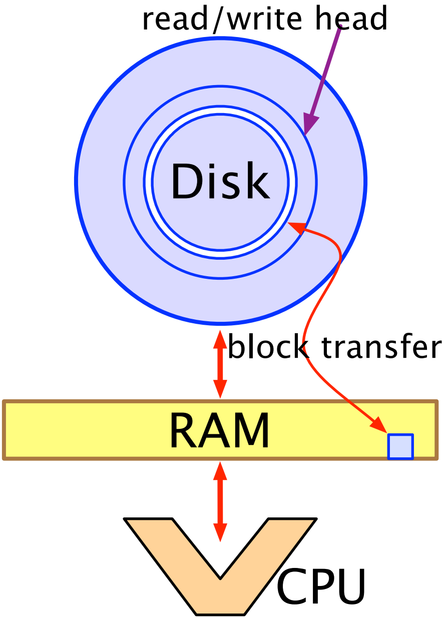
# Memory Hierarchy

- 1980s $\dashrightarrow$ 1990s Hierarchy expanded
- 1989: 486 processor has L1 Cache in CPU
  had L2 off CPU on motherboard
- L2 popular as motherboard speed rose



# Block Transfer

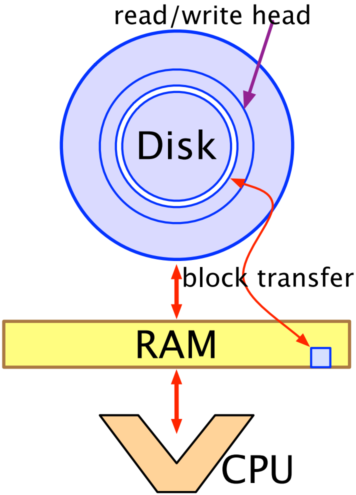
- Disk access is faster sequential: ($B$ = 8-16KB)
- Sends whole block to RAM (size $B$).
- RAM has size $M > B^2$.
- Disk access is $10^6$ more expensive than RAM access.
- Each block transfer is 1 I/O.
- Bound number of I/Os.



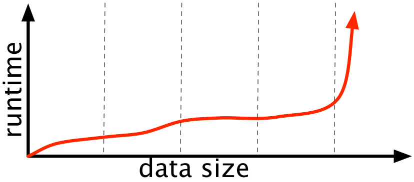
# Block Transfer

*The difference in time between modern CPU and disk technologies is analogous to the difference in speed in sharpening a pencil using a sharpener on one's desk or by taking an airplane to the other side of the world and using a sharpener on someone else's desk.*
- (Douglas Comer)

# Scalability




- Most programs developed in RAM model.
- Why don't they always thrash?



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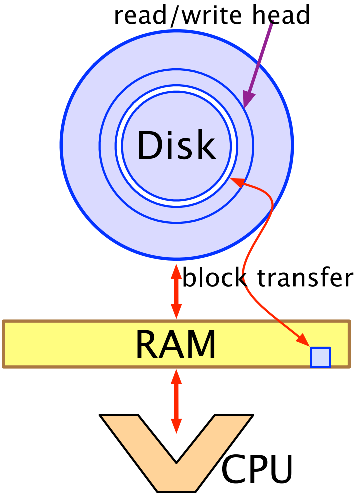


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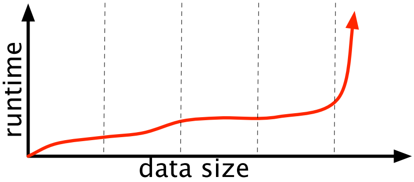


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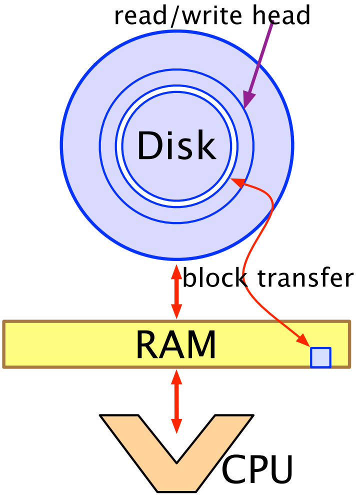


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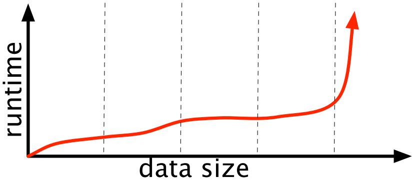


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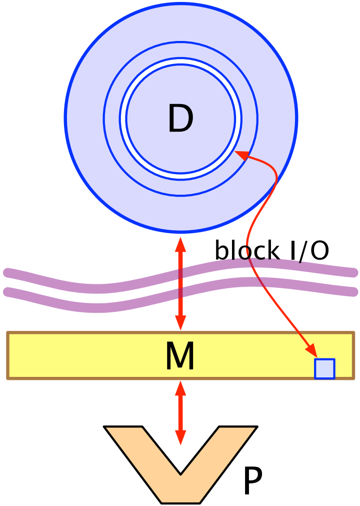
- Massive data and scattered access still spells doom.

# External Memory Model




- $N$ = size of problem instance
- $B$ = size of disk block
- $M$ = number of items that fits in Memory
- $T$ = number of items in output
- I/O = block move between Memory and Disk

[Aggarwal and Vitter '88]
[Floyd '72]

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- Linear I/O: $O(N/B)$
- Permuting not linear
- Permuting and sorting equal (practically)
- $B$ factor very important $\frac{N}{B} < \frac{N}{B} \log_{M/B} \frac{N}{B} \ll N$
- Cannot sort optimally with search tree

# Difference Between $N$ and $N/B$

Consider traversing a linked list.

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**Example**: $N = 256 \times 10^6$, $B = 8000$, 1ms disk access time

- $N$ I/Os takes $256 \times 10^3$ sec $= 4266$ min $= 71$ hours
- $N/B$ I/Os takes $256/8$ sec $= 32$ sec

# TPIE

Templated Portable I/O Environment
Open source library of I/O-Efficient data structures.

- External memory merge sort
- B-Tree
- Priority queue
- Simple buffered stacks and queues

`http://www.madalgo.au.dk/tpie/`

# Attribution

These slides are heavily based on slides by Lars Arge
(a leading expert in the area of External Memory algorithms).
See: `http://www.daimi.au.dk/~large/ioS09/`